PAP-003-1015001

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

October / November – 2018

Mathematics : Paper-5(A)

(Mathematical Analysis & Group Theory)

Faculty Code : 003
Subject Code : 1015001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : All questions are compulsory.

1 (A) Answer the following questions briefly : **4**

(1) Define closed set

(2) Give an example of a subset of metric space R which is not open and closed.

(3) If (X, d) is a discrete metric space and $a \in X$ then $N(a, 1/3) =$ ....

(4) Find the derived set of subset (1, 3) of metric space R.

(B) Attempt any one out of two : **2**

(1) If (X,d) is a metric space and $A, B \subset X$ then prove that $A^0 \subset B^0$.

(2) Obtain border set of subset (2,3) of metric space R.

(C) Attempt any one out of two : **3**

(1) Prove that arbitrary union of open sets of a metric space is open.

(2) Prove that every finite subset of metric space is closed set.

(D) Attempt any one out of two : **5**

(1) Prove that Derived set of any subset of a metric space is a closed set.

(2) If (X,d) is metric space and $E \subset X$ then E is open $\Leftrightarrow X - E$ is closed.

2 (A) Answer the following questions briefly : **4**

(1) Define partition.

(2) Define lower Riemann sum

(3) If P = {1,2,5,10} then find Norm of P

(4) Every constant function is Riemann Integrable. (True or False)

(B) Attempt any one out of two : **2**

(1) In usual notation prove that $L(P_1,F) \leq U(P_2,f)$.

(2) If $f(x)=x, x \in [0,1]$ and partition $P=\{0,1/3,2/3,1\}$ then find $U(P,f)$.

(C) Attempt any one out of two : **3**

(1) If f is continuous on [a,b] then prove that f is R-Integrable on [a,b].

(2) If f,g are R-Integrable on [a,b] then prove that f+g is also R-Integrable.

(D) Attempt any one out of two : **5**

(1) State and prove necessary and sufficient condition for a bounded function f defined on [a,b] to be R-Integrable.

(2) Find Riemann integral of $f(x)=x^2$ on [0,1] by second definition.

3 (A) Answer the following questions briefly : **4**

(1) Define Group

(2) Define integral function

(3) Define order of a group

(4) State second mean value theorem of Riemann integration

(B) Attempt any one out of two : **2**

(1) Prove that every element of a finite group is of finite order.

(2) State and prove first mean value theorem.

(C) Attempt any one out of two : **3**

(1) Prove that identity element is the only idempotent element in any group.

(2) Evaluate :

$$\lim_{n\to\infty} \frac{\pi}{n}\left[\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \ldots\ldots\ldots\ldots + \sin\frac{\pi\pi}{n}\right].$$

(D) Attempt any one out of two : **5**

(1) Prove that $\frac{\pi^2}{10} \le \int_0^{\pi} \frac{x}{3 - 2\cos x}\,dx \le \frac{\pi^2}{2}$.

(2) State and prove fundamental theorem of integration.

4 (A) Answer the following questions briefly : **4**

(1) Define Odd permutation.

(2) Define cyclic subgroup.

(3) Define centre of a group.

(4) Find the generators of cyclic group $(Z_5, +_5)$

(B) Attempt any one out of two : **2**

(1) If $\sigma = (1\ 2\ 3\ 4), \sigma \in S_5$ then find σ^{-1}

(2) Check whether $(Z, +)$ is cyclic group or not.

(C) Attempt any one out of two : **3**

(1) Prove that intersection of two subgroups of a group is also a subgroup.

(2) Let $H \le G$ and let $a, b \in G$ then show that

$$aH = Ha \Leftrightarrow ab^{-1} \in H \cdot$$

(D) Attempt any one out of two : **5**

(1) State and prove Lagrange's theorem for finite groups.

(2) If H and K are any two subgroups of group G such that (O(H,O(K)) = 1 then show That $H \cap K = \{e\}$.

5 (A) Answer the following questions briefly : **4**

(1) Define factor group.

(2) Define simple group.

(3) Define normal subgroup

(4) Every subgroup of a commutative group is normal. (True or False)

(B) Attempt any one out of two : **2**

(1) If H is a normal subgroup of group G with i_G (H) = m then prove that $a^m \in H; \forall a \in G$.

(2) If a finite group G has only one subgroup H of given order then H is a normal Subgroup of G.

(C) Attempt any one out of two : **3**

(1) If a subgroup H of a group G is normal subgroups of group G iff $aha^{-1} \in H; \forall a \in G, \forall h \in H$.

(2) Prove that a subgroup of index 2 in a group is a normal subgroup.

(D) Attempt any one out of two : **5**

(1) State and prove Cayley's theorem.

(2) Show that $(R+, \cdot) \cong (R, +)$.
